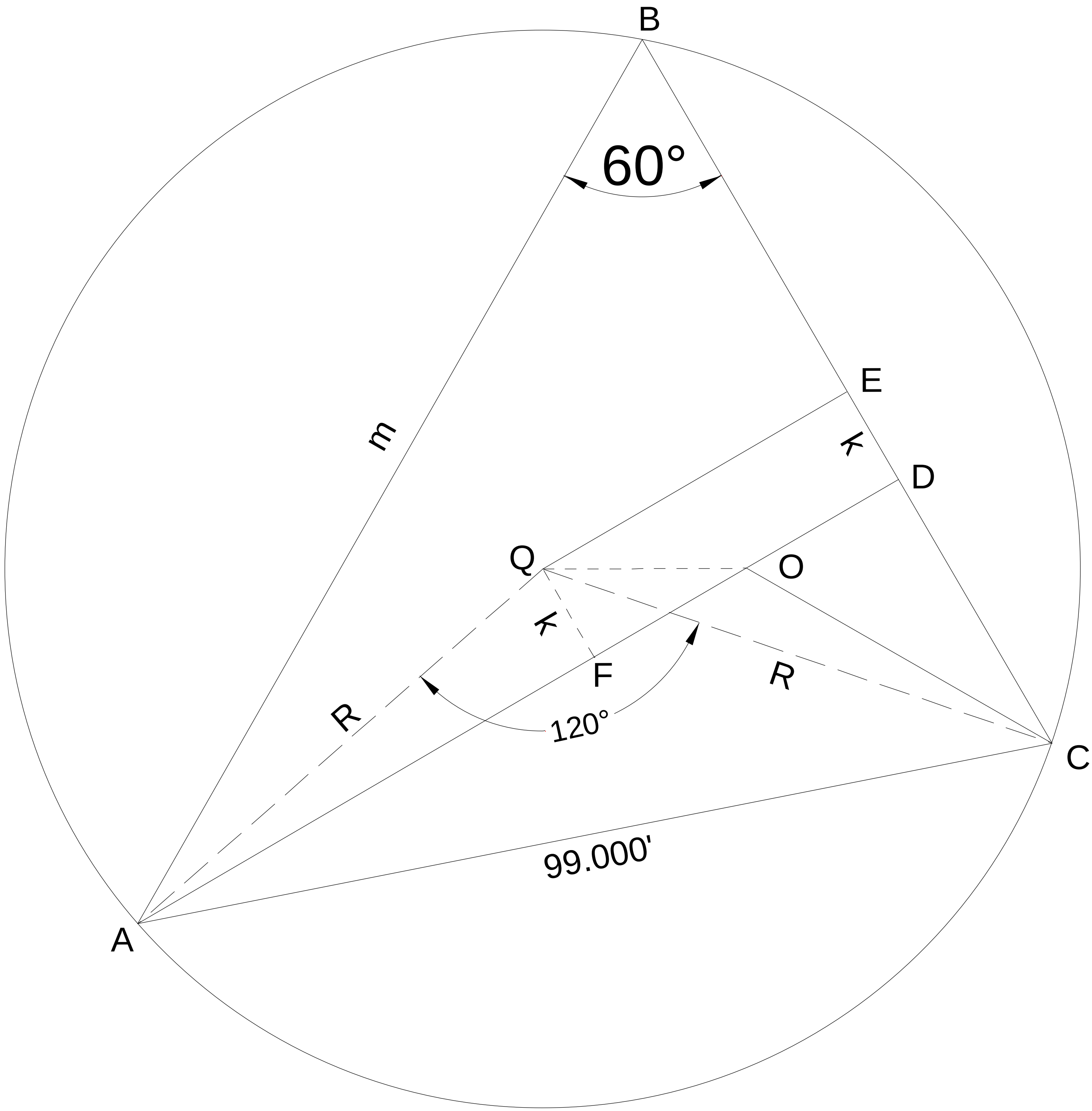
B
60°
E
k
D
m
Q
O
k
F
R
R
120°
C
99.000'
A

Construct the circumcircle with center at Q by intersection of the perpendicular bisectors of the sides of ABC. Draw the two radii AQ & QC = R. Connect Q and C.

From Ptolemy's Theorem, AC·QO + AQ·OC = AO·QC. Because angle AQC = 120° (twice angle ABC), AC =2·R· $\frac{\sqrt{3}}{2}$, and AQ = QC = R, so

$2\cdot R\cdot \frac{\sqrt{3}}{2}\cdot QO + R\cdot OC = AO\cdot R$, but AO = 2·OC

$\sqrt{3}\cdot QO + OC = 2\cdot OC$, and $OC = \sqrt{3}\cdot QO$ . . . . [1]

[R may be computed using triangle AQC and the Law of Cosines:

$99^2 = AQ^2 + QC^2 - AQ\cdot QC\cdot 2\cdot \cos 120° = R^2 + R^2 - 2\cdot R\cdot R(-0.5) = 3\cdot R^2$

From which R = $\sqrt{\mathbf{3}}(99)$ = 57.1577, but it is not needed for this problem]

Extend AO to intersect BC at D. Draw QE parallel with AOD and QF perpendicular to AO. Let AB = m and QF = k.

BE = EC because QE is the perpendicular bisector of BC. ABD is a 30°-60°-90° triangle, so BD = $\frac{m}{2}$, AD = $\frac{\sqrt{3}}{2}\cdot m$, and ED = QF = k.

DC = EC – k = BE – k = (BD-k) – k = BD – 2·k and EC = $\frac{m}{2} - 2\cdot k$

OD = $\frac{\left(\frac{m}{2}-2\cdot k\right)}{\sqrt{3}}$, OC =2· $\frac{\left(\frac{m}{2}-2\cdot k\right)}{\sqrt{3}}$, or OD = $\frac{m-4\cdot k}{2\cdot\sqrt{3}}$ and OC = $\frac{m-4\cdot k}{\sqrt{3}}$

From [1]: $\frac{m-4\cdot k}{\sqrt{3}} = \sqrt{3}\cdot OQ$, but OQ = 2·k, so m – 4·k = 6·k and m = 10·k

Now BD = 5·k, BE = 4·k, CD = 3·k

In triangle ADC, $AD^2 + DC^2 = 99^2$, or $\left(\frac{\sqrt{3}}{2}\cdot m\right)^2 + (3\cdot k)^2 = 99^2$

$\left(\frac{\sqrt{3}}{2}\cdot 10\cdot k\right)^2 + 9\cdot k^2 = 99^2$ from which k = 10.8018

AB = 10·k = 108.0178 and BC = 8·k = 86.4143

$\text{Cos A} = \frac{108.0178^2+99^2-86.4143^2}{2\cdot 108.0178\cdot 99}$ = 0.654653321 = cos 49°06'24"

$\text{Cos C} = \frac{99^2+86.4143^2-108.0178^2}{2\cdot 99\cdot 86.4143}$ = 0.327327636 = cos 70°53'36"