

Low Distortion Projections—Part 1

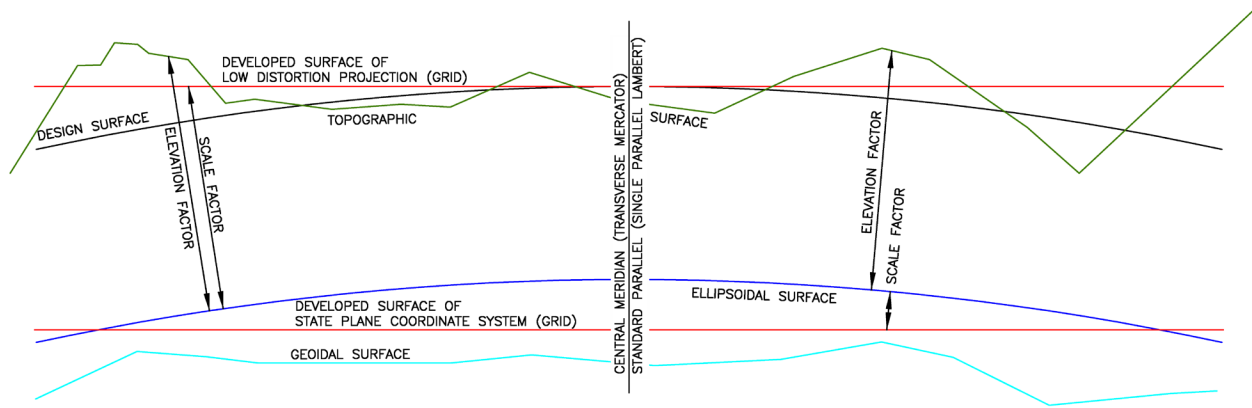
GROUND VERSUS GRID

According to an interesting read entitled *Geodetic Surveys in the United States: The Beginning and the Next 100 Years*¹ and in a letter from January of 1937, presented by the Federal Board of Surveys and Maps², the State Plane Coordinate System was developed in the early 1930's by a committee led by Oscar S. Adams of the U.S. Coast and Geodetic Survey to make geodetic surveying with the shiny new NAD27 adjusted geodetic network (previously only provided in latitude and longitude) more accessible to the common surveyor and engineer. Initially, two conformal projections were employed: Lambert for States, or subdivided zones,

predominantly oriented East to West, and Transverse Mercator for States, or subdivided zones, predominantly oriented North to South. Furthermore, the individual projections were to have a scale factor of no more than 1:10,000 (except for North Carolina), which could also be expressed as 100 ppm. This scale factor should not be confused with the combined factor (which includes elevation factors) which can easily exceed this arbitrary threshold (and often does).

With thousands of adjusted stations published in latitude and longitude all over North America, what could a surveyor do with them without learning and applying spherical trigonometry? The need for the SPCS was great and its development

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In this illustration, the five surfaces effecting projection design are depicted: Geoidal, Ellipsoidal, Design, Developed, and Topographic). Both Low Distortion and State Plane Grids are represented. Notice that the Elevation Factor would be the same for both projection types, however because the State Plane Grid is designed to be near the Ellipsoidal surface, the State Plane Grid scale factor will result in Grid distances always being relatively close to Ellipsoid distances, however, the Low Distortion Projection Scale Factor offsets the Elevation Factor giving a distance that is very close to the Design surface (an elevated Ellipsoid) and, thus, if designed correctly, will be very close to horizontal surface distances. Also note, that in this example, the LDP Grid is always above the design surface, but as Loyal Olson points out, by intentionally lowering the LDP Grid below the design surface (by subtracting a single part-per-million from our design scale factor) we can extend the useful range of our LDP by several miles (see ppm chart).

nothing short of ingenious. For the first time ever, surveyors and engineers with limited computational power could extend geodetic control from a published monument to any project within feasible range with little modification to their normal field methods, all on a nationally recognized grid system. Of course, at this time there were no electronic computers or electronic calculators. In fact, illustrating the limited computational power available to the practicing surveyor, it would be more than a decade before the Jewish prisoner, Curt Herzstark, could manufacture his portable, hand-cranked Curta calculator. The commonly available calculating tools of the day were slide rules and logarithmic tables which were

used to reduce latitudes and departures, and the SPCS was carefully crafted to be functional within that framework.

Per the letter by the Federal Board of Surveys and Maps², Adams' considered opinion was that the inconvenience of dealing with potentially large combined factors (elevation factor x scale factor) was trivial compared to the benefit surveyors and engineers could realize tying their cadastral work to a nationally recognized planar coordinate system, and he shared users' opinions that supported the assertion. So long as users appropriately applied the scale factors to measured distances, the system worked well. However, soon after the State Plane Coordinate System was developed, departments

of transportation and other users with large projects across the country began bastardizing the system to address these combined factors by improperly applying scale factors to coordinates rather than distances. Incidents of "modifying" the SPCS only increased as a result of easier connections to geodetic monuments via differential GPS and easier computations via the modern computer. But time has shown that, whether by unintentional ignorance or by willful laziness, too many surveyors have employed shoddy, short-cut solutions to rectify scale factors in their work, virtually gutting the perceived reliability of the SPCS.

Are you following a survey that gives State Plane Coordinates to monu-

1ppm	21 feet	11.2 miles
2ppm	42 feet	15.9 miles
3ppm	63 feet	19.4 miles
4ppm	84 feet	22.4 miles
5ppm	105 feet	25.1 miles
6ppm	126 feet	29.7 miles

This chart depicts the changes in the elevation factor or scale factor at various positions related to the Design Surface or Developed Surface. The first column is the parts per million (ppm) change and represents a change to elevation factors with respect to the second column or a change to the scale factor with respect to the third column. The second column represents a change in ellipsoid height. Thus for an 84 foot change in ellipsoid height, there is a 4ppm change to the elevation factor. The third column represents the total width within a projection that would maintain a scale factor at or less than the corresponding scale factor in the first column. Thus the difference between all of the scale factors within 9.7 miles East or West from the central meridian of a Transverse Mercator projection or North or South from the Standard Parallel of a Single Parallel Lambert projection would less than 3 ppm, providing a total width of 19.4 miles (9.7miles x 2).

ments and those coordinates appear to be hundreds of feet from where they should be? The surveyor who published those coordinates likely applied a scale factor to their coordinates from an origin of 0,0. If the given State Plane Coordinates of a survey all appear to be *pretty good* over most of the project and really good in a specific area, the coordinates were likely scaled from the true State Plane Coordinates of a particular point in the project. The coordinates in the first example are all but useless, while in the second, the coordinates are only marginally faithful to the system they purportedly represent.

The SPCS isn't to blame for these violations. If a surveyor carefully accounts for the scale factors and convergence angles, the SPCS is perfectly precise. Only when these distortions are ignored or mishandled do problems appear. Just perhaps, however, the poor remedies that have been applied for decades to these distortions, which are so commonly exercised, suggest some deficiencies in the system itself. If the scale factors weren't so excessive, per-

history working in the rugged Mountain West. He's been using LDP's in his work for decades, originally using software written for a Wang computer in the 1970's. Loyal's arguments for LDP's and his own personal experiences working with them convinced me that LDP's were worth considering. He has provided volumes of support to me as I have felt my way through the design and implementation of Low Distortion Projections these past seven years. In recent months, we've been reacquainted through the Surveyor Connect message board and he continues to be an extremely helpful advocate of the concept to those wanting to understand LDP's. The significance of his input in this article cannot be overstated. Even more, Loyal has not only used LDP's in a textbook methodology, he's also employed them in creative ways to handle more unconventional situations which I'll describe later.

The Low Distortion Projection is a concept promoting precisely what the name implies. All projections introduce

North, and that can be used in an RTK controller, GNSS post processing software or mapping software to directly convert from those coordinates to latitude and longitude or any other projection (such as State Plane or UTM) at the touch of a button.

Increasingly, software developers are supporting custom projections in their products. All of the RTK controllers I've used in the past few years support custom projections. Post processing GPS software has supported custom projections for years. Numerous desktop COGO/CAD software providers currently support custom projections (including Carlson Software, MicroSurvey and Autodesk). I began working seriously with LDP's when North Carolina surveyor, Chuck Rushton, gave me a program for the HP-49G+/HP-50g calculator that could do conversions and inverses of coordinates in various predefined and user defined projections. His SPC50 program is incredibly intuitive for a calculator based program and handles user defined projections on a very inexpensive and portable platform. For readers interested in giving SPC50 a try, Chuck tells me he can be reached by email at cwrpls@bellsouth.net. As helpful as these various offerings are, perhaps the king of coordinate transformation software and manipulation is Blue Marble Geographics' Blue Marble Desktop, which allows for transformation of not just coordinate points and coordinate files, but also of raster graphics (sid, tif, jpg, bmp, etc.) and vector files (dgn, dwg, dxf, etc.) Using Blue Marble Desktop means you can transform a coordinate point, a coordinate file, an aerial image, or a CAD drawing from any projection to any other projection or system with ease. The aerial you have from your State's website in UTM can be reprojected into SPC or your own LDP. Last year's CAD drawing done in its own projection can be reprojected with this year's project half a mile away into a different projection. I've been working with Blue Marble Desktop for several months now and I'm continually impressed by the power and simplicity it

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haps surveyors and engineers wouldn't feel so compelled to compensate for them in this manner. After all, wouldn't it be nice if the inverse between your georeferenced coordinates matched the measurement you made on the ground? Perhaps if the convergence angles weren't so extreme, surveyors would be satisfied to leave North alone. Wouldn't it be nice if North was very near actual (dare I say True) North throughout your survey? While the developers and employers of the State Plane Coordinate System did not have an easy alternative to these issues because of technical challenges, we do today as a result of the incredible computing power now available to us. That alternative is the Low Distortion Projection—LDP.

A few years ago, through the old RPLS.com message board, I “virtually” met Loyal Olson, a surveyor with a long

distortions. Ever since Pythagoras declared the Earth was round, flat maps of a curved surface have confounded humanity. The larger the area covered by a projection, the greater the distortions. The LDP development process looks to minimize those distortions by custom fitting a projection to a project.

The projection may cover a county or city, a section or original grant, a neighborhood or even various elevation zones of any of the above. The projection may look to have North be North along some particular meridian, or be customized to fit an existing bearing in an existing coordinate system that was originally determined by less precise means (such as from a compass). The “plus” to all of this is a system that gives coordinate inverses substantially equivalent to distances measured along the ground, and is related to North or very nearly

brings to working with various projections and file types in a very mature and stable application.

So I've laid out the problem and offered a solution. The problem is that we are forcing an analog solution to continue to work in a digital world. As brilliant as the development of the SPCS is, the misuse and abuse over these past decades suggests it is deficient for modern application. At least one credible solution is the use of custom, Low Distortion Projections. Modern software, available in the field and in the office, can easily clear the calculation hurdles encountered with LDP's. The only issue left is the depth of the knowledge well of the professional surveyor that will be applying them. A little knowledge about projections and some practice with them should leave him ready to implement them in his workflow. So let's get practical and explore some of the basic elements of a projection and put a couple into practice on a live exercise.

As surveyors working with projections, we want to minimize distortions to our field observed measurements—angles and distances. “Conformal” projections limit the amount of error in angles turned on the ground over moderate distances to a negligible amount. We like this situation because, with a conformal projection selected, our angles remain unchanged and now our only concern is in the magnitude of the scale factors applied to our measured distances. While there are more projection types than I will ever know or comprehend, I work with only two projection types, and could probably work with only one for all of the projects we have: the first is the Lambert Single Parallel and the second is the Transverse Mercator. Both are conformal, and both have identical user defined elements. The Transverse Mercator is more commonly supported, making it a favored choice when developing an LDP, however all of the desktop survey software I own as well as all of

the RTK controller software I've used in the past few years have supported both projection types.

Another projection type that is less supported than Transverse Mercator and Lambert Single Parallel, and somewhat more complicated than both is the Oblique Mercator. While it is beyond the scope of this article to delve into the Oblique Mercator projection type, it is worth mentioning because of the unique applications it addresses. Lambert projections minimize the magnitude of scale distortions East and West from the origin, while Transverse Mercator projections minimize the magnitude of scale distortions North and South from the origin. Oblique Mercator projections minimize the scale distortions along a defined azimuth for projections that don't fit well with predominantly cardinal orientations. Oblique Mercator projections also allow

cone touches the basketball is the Standard Parallel for a Single Parallel projection type. The scale factor is exactly 1 along this ring of latitude. No matter how far East or West you run along that latitude, the scale factor remains the same. The further North or South along the basketball you go from this ring, the distances along the cone and the distances along the ball become increasingly less similar producing an increased scale distortion. Thus, the Lambert projections are more suited to projects with an East-West extent, as the scale factors will be more limited in magnitude.

For the Transverse Mercator projection, we'll need a baseball and a wide mouth coffee cup. Place the ball in the cup. Place the cup on its side. You now have a model of the Transverse Mercator projection. The ball touches the cup along a line of longitude, or meridian.

“Manipulating the Design Surface is the key to reducing the extreme combined factors found in large projections and provide us with the ‘low distortion’”

users a unique solution to fitting a projection to an existing coordinate system with a bearing basis that deviates from geodetic North—such as systems originally based on magnetic North or an assumed bearing. Loyal has used this projection type to accommodate mine grids and even old Town/City grids that were rotated tens of degrees from “North” and expressed at elevations ranging from 5,000 feet to 10,000 feet. Using the Oblique Mercator projection in this context allowed him to effectively relate historic coordinate values to modern NAD83 Geodetic values.

Visualizing the projections will help us to better understand the nature of the scalar distortions each present. Take a traffic cone and set it on a basketball. Congratulations, you have demonstrated a Lambert projection Single Parallel projection. The theoretical circle where the

Along this meridian line the scale factor is exactly one and remains constant along that longitude. Go East or West from that longitude and the scale factor changes, as the distance along the cup and the distance along the ball correspond less and less. Therefore in projects with a predominantly North-South extent, the scale factors are less affected.

As for the convergence angle (aka mapping angle, gamma angle, theta angle) remember, we are maintaining our turned angles on the ground and we work on a round Earth. Regardless of the conformal projection used, mapping angles will be part of the process. As one moves further from the central meridian of the projection, the magnitude of the convergence angle will increase.

As we discuss scale factors, it will probably become necessary to get a clear picture on why we have them. In order to do this, we will need to visualize the

various surfaces on which we work. For Geodetic purposes, we work with 5 different surfaces:

ELLIPSOIDAL SURFACE

The first is the Ellipsoid. So the Earth isn't perfectly shaped, but the best, simplest, mathematical form used to model her overall form is the Ellipsoid. Take a proper oblate ellipse and rotate it around its minor axis and you have a fair representation of the shape of the Earth.

TOPOGRAPHIC SURFACE

The second is the Topographic Surface. The Topographic Surface of the Earth is craggy, hilly, deep, high, low, etc. There is no simple model for these undulations which is what keeps us, as surveyors, in business by meeting the ever-constant need to map its unwieldy shape. For mathematical simplicity Geodetic coordinate calculations and inverses are performed on the ellipsoid, however, because the ground surface and the ellipsoid are different (and often substantially so), we have a correction factor that needs to correlate the two. This factor is commonly referred to as the elevation factor. It will vary with variations in the ellipsoid height. Because the Earth is really big, and the changes between the two surfaces are generally small by comparison, a single elevation factor can often be applied to a project. Exceptions might be found in mountainous terrain with large variations in the ellipsoid height across a project.

GEOID

The third surface is the Geoid. The Geoid considers the affect gravity would have on sea level. In the big adjustment of NAD27, Geoid issues began to be noticeable. A star shot on the side of a mountain was affected by the mountain because the level on the theodolite didn't point parallel to the ellipsoid, it was perpendicular (or normal) to the downward pull of gravity. Mountains are big and have lots of mass, and where there is mass there is gravity. So in the next big adjustment,

NAD83, the effect of gravity at all 193,000+ stations was considered³. For our purposes as surveyors, a more easily digested explanation of the equipotential surface represented by a geoid would be to consider the geoid as mean sea level (unaffected by tide or wind) if it were brought to your project. This would result in the water surface forming peaks at places of higher gravitational pull (i.e. more mass), particularly mountains.

DESIGN SURFACE:

This concept may be somewhat foreign to flat land surveyors, however, surveyors familiar with working in mountainous terrain are well acquainted with this surface. For a long time surveyors working in these places have understood that closures on their surveys suffered unless each distance was scaled to a common ellipsoid height, or design surface. This is particularly true of surveys that may involve thousands of feet in elevation difference. Distances measured between points below the design surface are scaled up by the elevation factor, and those measured between points above the design surface were scaled down by the elevation factor. This relative elevation factor is determined by:

$$\frac{\text{Design Ellipsoid Height} + \text{Mean Earth Radius}}{\text{Point Ellipsoid Height} + \text{Mean Earth Radius}}$$


Keep in mind that although the term "Elevation Factor" might suggest an orthometric height, this factor is actually based on ellipsoidal heights. In a project with an extreme elevation difference of 2000 feet, and a Design Surface ellipsoid height of 9000 feet, a point at 8000 feet will have a relative elevation factor of 1.0000478, and a point at 10,000 feet will have a relative elevation factor of 0.9999522. As can be seen from these values, ignoring the differences could substantially affect traverse closures.

For the purposes of our Low Distortion Projection design, manipulating the Design Surface is the key to reducing the extreme combined factors

found in large projections, such as the State Plane Coordinate System and Universal Transverse Mercator, and provide us with the "low distortion".

DEVELOPED SURFACE (GRID)

The fourth surface is our developed surface or projection Grid. Like the ellipsoid, it is purely mathematical. It has no error (note that the SPCS's maximum 1:10,000 scale factor is not an error, it is a distortion that can be precisely determined for any point in the projection). The grid may be above the design surface, in which the scale factor will be above 1, or the grid may be below the design surface, in which the scale factor would be less than 1. This scale factor, often represented by the letter *k*, is independent of the elevation factor, but it can be manipulated in our projection design to provide a project wide combined factor (elevation factor x scale factor) approaching 1.

In Part 2, I'll share two real world Low Distortion Projections and the process I used to design them as well as some practical reasons for implementing your own custom projections in your work. 

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FOOTNOTES

¹ www.ngs.noaa.gov/PUBS_LIB/geodetic_survey_1807.html

² www.ngs.noaa.gov/PUBS_LIB/SPCUSCGSAdams1937.pdf

³ www.ngs.noaa.gov/PUBS_LIB/NADof1983.pdf