



In general, a third circle will cut two others orthogonally simultaneously if the tangents from a point, P, to both circles are equal. P lies on the *radical axis* of circles Q and R and line PS is perpendicular to the line joining the circle centers, refer to Figure 1:

$$\overline{PT}^2 = \overline{PQ}^2 - r_1^2 = \overline{Pt}^2 = \overline{PR}^2 - r_2^2$$

$$\overline{PS}^2 + m^2 - r_1^2 = \overline{PS}^2 + n^2 - r_2^2$$

$$m^2 - r_1^2 = n^2 - r_2^2$$

$$m^2 - n^2 = r_1^2 - r_2^2$$

Let $m + n = d$, the distance between circle centers, then

$$(m + n)(m - n) = r_1^2 - r_2^2$$

$(m - n) = \frac{r_1^2 - r_2^2}{d}$ along with $m + n = d$ is easily solved for m and n:

$$m = \frac{d}{2} + \frac{r_1^2 - r_2^2}{d}, \quad n = \frac{d}{2} - \frac{r_1^2 - r_2^2}{d}$$

For the radical line of circles A and B:

$$m = \frac{559.521}{2} + \frac{267.625^2 - 214.017^2}{559.521} = 302.8337$$

$$n = \frac{559.521}{2} - \frac{267.625^2 - 214.017^2}{559.521} = 256.6873$$

For the radical line of circles A and C:

$$m = \frac{488.622}{2} + \frac{267.625^2 - 137.966^2}{488.644} = 298.1327$$

$$n = \frac{488.644}{2} - \frac{267.625^2 - 137.966^2}{488.644} = 190.5113$$

And, as a check, the radical line for circles B and C:

$$m = \frac{447.038}{2} + \frac{214.017^2 - 137.966^2}{447.038} = 253.4390$$

$$n = \frac{447.038}{2} - \frac{214.017^2 - 137.966^2}{447.038} = 193.379$$

Calculating coordinates for points a, b, and c along the lines joining radius points gives:

a= N:1163.250, E:1074.880; b= N:935.046, E:1097.106 and c= N:1039.845, E:866.080

Perpendicular intersections give Q = N: 1043.440, E: 1000.879

The radius of circle Q = $\sqrt{AQ^2 - 267.625^2} = \sqrt{331.499^2 - 267.625^2} = 195.624$