

EQUAL CHORD SOLUTION

Calculate the sides of triangle ABC: $AB = \sqrt{13}, BC = \sqrt{17}, AC = \sqrt{8}$

The angles of triangle ABC are:

$$\cos A = \frac{(\sqrt{13})^2 + (\sqrt{8})^2 - (\sqrt{17})^2}{2 \cdot \sqrt{13} \cdot \sqrt{8}} = 78^\circ 41' 24.2''$$

$$\cos B = \frac{(\sqrt{13})^2 + (\sqrt{17})^2 - (\sqrt{8})^2}{2 \cdot \sqrt{13} \cdot \sqrt{17}} = 42^\circ 16' 25.3''$$

$$\cos C = \frac{(\sqrt{8})^2 + (\sqrt{17})^2 - (\sqrt{13})^2}{2 \cdot \sqrt{8} \cdot \sqrt{17}} = 59^\circ 02' 10.5''$$

In triangle RAC, $\cos ARC = \frac{5^2 + 5^2 - (\sqrt{8})^2}{2 \cdot 5 \cdot 5} = \cos 32^\circ 51' 35.6''$ making angles RAC and

RCA = $73^\circ 34' 12.2''$

The bearing of line AB = $\arctan 3/2 = N 56^\circ 18' 35.8'' E$, which makes line AC $180^\circ - 56^\circ 18' 35.8'' - 78^\circ 41' 24.2'' = S 45^\circ 00' 00.0'' E$, which checks its coordinates. Line AR is then $S 28^\circ 34' 12.2'' W$. Angle EAR which equals angle REA = $56^\circ 18' 35.8'' - 28^\circ 34' 12.2'' = 27^\circ 44' 23.6''$ so angle ERA = $124^\circ 31' 12.8''$ and EA = $2 \cdot 5 \cdot \sin 62^\circ 15' 36.4'' = 8.8507$ and point E is $S 56^\circ 18' 35.8'' 8.8507$ from point A, or $(-2.3642, 5.0905)$.

Starting again from line CA being $N 45^\circ 00' 00.0'' W$, line CB is $N 14^\circ 02' 10.5'' E$ and line RC is $S 61^\circ 25' 47.8'' W$ so angle RCD = angle RDC = $47^\circ 23' 37.3''$ and angle DRC = $85^\circ 12' 45.4''$ and CD = $2 \cdot 5 \cdot \sin 42^\circ 36' 22.7'' = 6.76957$, making point D $S 14^\circ 02' 10.5'' W 6.76957$ from C, or $(5.3581, 1.4326)$ and therefore DE

$$= \sqrt{[5.3581 - (-2.3642)]^2 + [1.4326 - 5.0905]^2} = 8.5448$$

With point B moving to B' (8.883, 11.214), the angles of triangle AB'C are

A = $62^\circ 21' 41.7''$, B' = $42^\circ 16' 24.5''$, and C = $75^\circ 21' 53.7''$

Angle RCD' = $180^\circ - 75^\circ 21' 53.7'' - 73^\circ 34' 12.2'' = 31^\circ 03' 54.1''$ and angle CRD = $117^\circ 52' 11.8''$. Angle E'AR which equals angle RE'A = $180^\circ - 62^\circ 21' 41.7'' - 73^\circ 34' 12.2'' = 44^\circ 04' 06.1''$ so angle E'RA = $91^\circ 51' 47.8''$ and angle E'RD' = $117^\circ 24' 24.8''$ (The first solution could have been done using this technique, too, instead of calculating coordinates) and E'D' = $2 \cdot 5 \cdot \sin 58^\circ 42' 12.4'' = 8.5449$, the same as ED, except for rounding errors.

In fact, wherever point B moves along the circle between A and C, the resulting chord ED will be the same length.

