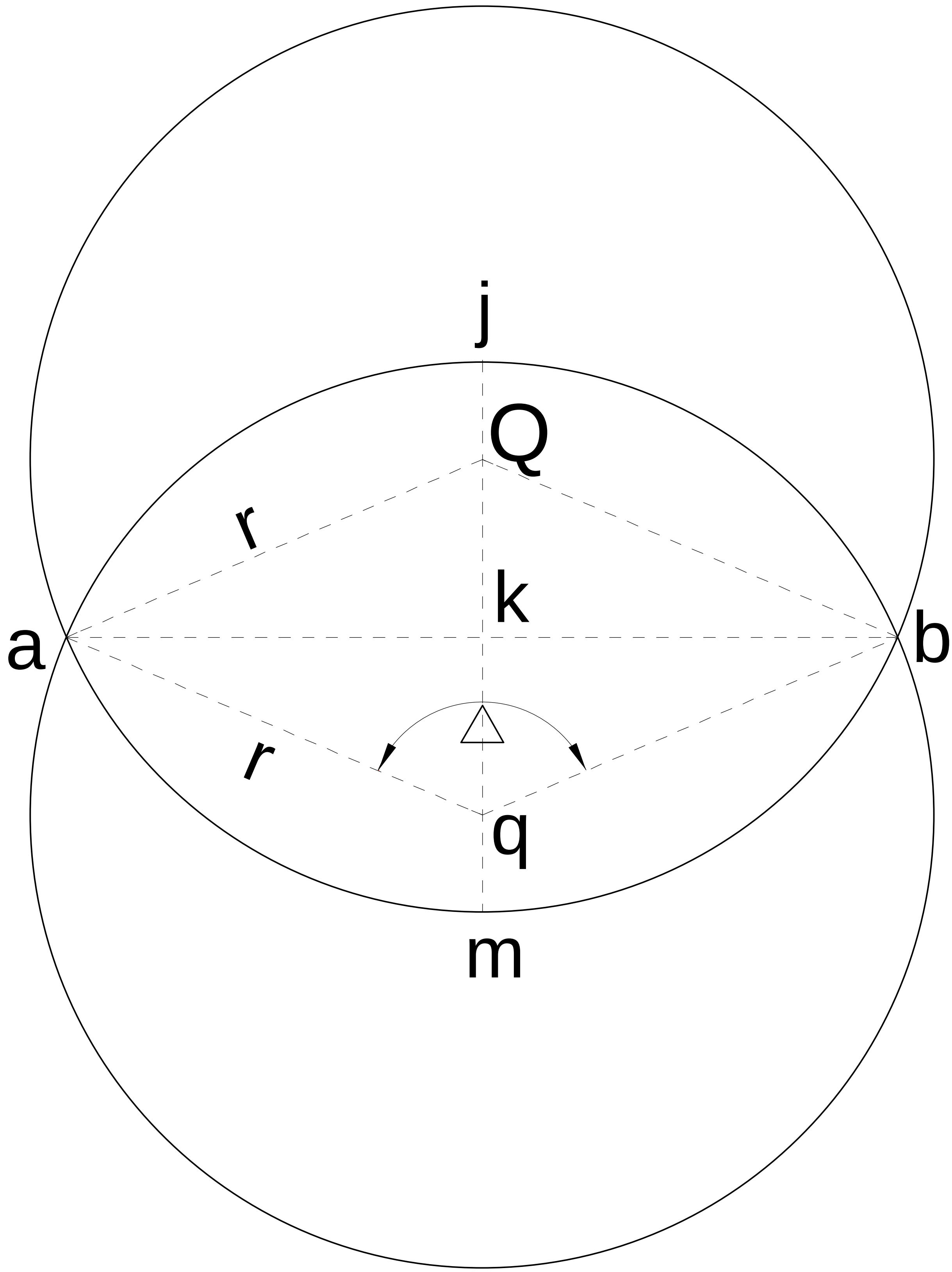
3 AREAS SOLUTION



3AreaAns

For each lune to have the same area they must each be one-half the area of a full circle, πr^2.

½ area A = ½ area B = ½ area C = ¼ the area of a full circle, or $\frac{\pi \cdot r^2}{4}$. The area of sector abq = $\frac{\triangle}{360} \cdot \pi \cdot r^2$, the area of triangle abq = $\frac{1}{2} \cdot \sin\triangle \cdot r^2$

$$\frac{\triangle \cdot \pi \cdot r^2}{360} - \frac{\sin\triangle \cdot r^2}{2} = \frac{\pi \cdot r^2}{4} \quad \text{or} \quad \frac{\triangle \cdot \pi}{180} - \sin\triangle - \frac{\pi}{2} = 0$$

(the first term actually gives the central angle in degrees expressed as radians)

Δ must range from 0° to 180°. Using only a ten digit calculator and starting at 90°, or 1.570796327 radians, the sin value is 1.000… and the equation value is –1.000…

Trying 110° (1.919862177 radians) the sinΔ = 0.939692621 and the equation value is –0.590626770, which appears to be the correct direction to go (toward zero).

For 130° (2.268928028 radians), sinΔ = 0.766044443 and the equation value is –0.067912742

For 150° (2.617993878), sinΔ = 0.5…, and the equation value is 0.547197551, thereby narrowing the range of Δ from 130° to 150°. Straight linear interpolation yields 132.2081484° (2.307467488 radians) with sinΔ= 0.740709059 and an equation value of –0.004037898. Using that value and the 130° value closest to it, linear interpolation gives 132.3477382° (2.309903789 radians) and sinΔ= 0.739070092 for an equation value of 0.000037371. One more interpolation using the last two values gives 132.3464577° (2.309881441 radians), sinΔ = 0.739085147 and an equation value of –0.000000033, with a final answer of 132.3464588°

Angle aqb is 132°20'47.25" and akb=Y=$2 \cdot r \sin\frac{\triangle}{2} = 2(0.9144771014) = 1.829542 \cdot r$

Qq=$2 \cdot r \cos\frac{\triangle}{2} = 2r(0.403972762) = 0.807945525 \cdot r$

jkq = Qkm = r; jQ = qm = r – Qq = 0.192054475$\cdot$r, so X = 1.192054475$\cdot$r