"Test Yourself"

Prove It: Geometry v. Trigonometry

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In geometry, a theorem says: For two intersecting chords, the product of the lengths of the two segments of one chord is equal to the product of the lengths of the two segments of the other chord. Or, for the figure, (AX)(BX)=(CX)(DX). Ok, you geometry and horizontal circular curve buffs, prove it, each in your own way.

For the solution to this problem (and much more), please visit our website at: www.TheAmericanSurveyor.com Good luck!

Answers to "Test Yourself"

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The Geometry Person Would Say:

1.) Angles CXA and BXD are equal.

2.) Angles ACD and ABD are equal. (You highways surveyors will recognize that as being correct.)

3.) Therefore, triangles AXC and DXB are similar (same proportion).

4.) Therefore,

$$\frac{AX}{CX} = \frac{DX}{BX}$$

and, therefore,

$$(AX)(BX) = (CX)(DX)$$

Proved!

The Highway Surveyor Horizontal Circular Curve and Trigonometry Person (who doesn't like geometric proofs) Would Say:

1.) OK, for some R, let's give Points C, B, D and A some stationing. (Any reasonable numbers assigned will work.)

2.) For the two arc lengths, CD and BA (which will be the stationing differences), compute their two Deltas.

$$Delta = \frac{360° L}{2\pi R}$$

3.) By whichever method you wish, compute coordinates for C, B, D and A.

4.) As a check, using the coordinates, inverse to compute the chord distances CD and BA. Do they check with $LC=2R\sin\Delta/2$?

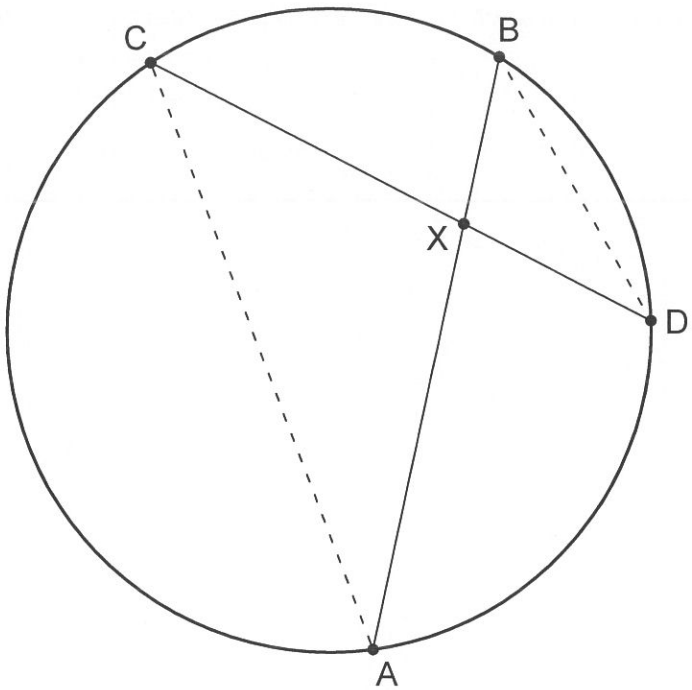
5.) Using the coordinates for the endpoints of the chords, do an intersection and compute the coordinates of X.

6.) Using the coordinates, inverse to find distances CX, DX, BX and AX.

7.) Finally, compare the products of (AX)(BX) and (CX)(DX). Are they equal (allowing for round off)?

8.) If they are not equal, the Geometry Person would say you have something wrong in your calculations. I would agree.

TY E29



(AX)(BX) = (CX)(DX)

PROVE IT